

O LEVEL (P2)

Graphical Solutions

QUESTION'S

1 Answer the whole of this question on a sheet of graph paper.

The table gives the x and y coordinates of some points which lie on a curve.

x	1	1.5	2	2.5	3	4	5	6
y	140	110	100	98	100	110	124	140

- (a)** Using a scale of 2 cm to represent 1 unit, draw a horizontal x -axis for $0 \leq x \leq 6$.

Using a scale of 2 cm to represent 10 units, draw a vertical y -axis for $90 \leq y \leq 150$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

- (b)** Use your graph to find

(i) the value of y when $x = 4.5$, [1]

(ii) the values of x for which $y = 128$. [1]

- (c)** By drawing a tangent, find the gradient of the curve at the point where $x = 1.5$. [2]

- (d)** The line $y = k$ is a tangent to the curve.

Find the value of k . [1]

- (e)** The values of x and y are related by the equation

$$y = \frac{A}{x} + Bx.$$

- (i)** Use the fact that the point (2, 100) lies on the curve to show that

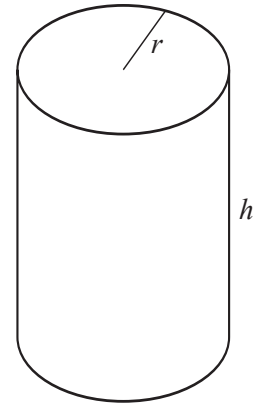
$$200 = A + 4B. \quad [1]$$

- (ii)** Obtain a second equation connecting A and B .

Hence calculate the value of A and the value of B . [3]

2 Answer the whole of this question on a sheet of graph paper.

A solid cylinder of radius r centimetres and height h centimetres has a volume of $100\pi \text{ cm}^3$.



(a) (i) Show that $h = \frac{100}{r^2}$. [1]

(ii) The cylinder has a total surface area of πy square centimetres.

Show that $y = 2r^2 + \frac{200}{r}$. [1]

(b) The table below shows some values of r and the corresponding values of y , correct to the nearest whole number.

r	1	1.5	2	3	4	5	6
y	202	138	108	85	82	90	p

(i) Find the value of p . [1]

(ii) Using a scale of 2 cm to represent 1 cm, draw a horizontal r -axis for $1 \leq r \leq 6$. [1]
 Using a scale of 2 cm to represent 20 cm^2 , draw a vertical y -axis for $70 \leq y \leq 220$.
 On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to find the values of r for which $y = 100$. [2]

(d) By drawing a tangent, find the gradient of the graph at the point where $r = 2$. [2]

(e) Use your graph to find

(i) the value of r for which y is least, [1]

(ii) the smallest possible value of the total surface area of the cylinder. [1]

3 Answer the whole of this question on a sheet of graph paper.

During one day, at a point P in a small harbour, the height of the surface of the sea above the seabed was noted.

The results are shown in the table.

Time (t hours) after 8 a.m.	0	1	2	3	4	5	6	7	8	9
Height (y metres) above the sea-bed	3.8	3.3	2.5	1.8	1.2	1.0	1.2	1.8	2.5	3.3

- (a) Using a scale of 1 cm to represent 1 hour, draw a horizontal t -axis for $0 \leq t \leq 9$.

Using a scale of 2 cm to represent 1 metre, draw a vertical y -axis for $0 \leq y \leq 4$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

- (b) (i) By drawing a tangent, find the gradient of the curve at the point where $t = 4$. [2]

(ii) Explain the meaning of this gradient. [1]

- (c) On the same day, a straight pole was driven vertically into the seabed at the point P .
Work started at 8 a.m.
The pole was driven in at a constant rate.
The height, y metres, of the top of the pole above the seabed, t hours after 8 a.m., is given by the equation

$$y = 4 - \frac{1}{2}t.$$

(i) Write down the length of the pole. [1]

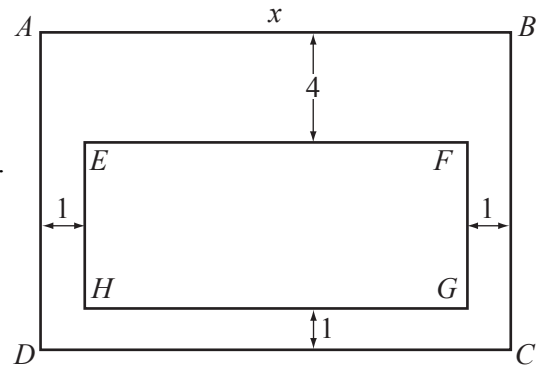
(ii) On the same axes as the curve, draw the graph of $y = 4 - \frac{1}{2}t$. [2]

(iii) How many **centimetres** was the top of the pole above the surface of the sea at noon? [2]

(iv) Find the value of t when the top of the pole was level with the surface of the sea. [1]

4 Answer the whole of this question on a sheet of graph paper.

The area of a rectangular garden, $ABCD$, is 100 m^2 .
 Inside the garden there is a rectangular lawn, $EFGH$,
 whose sides are parallel to those of the garden.
 EF is 4 m from AB .
 FG , GH and HE are 1 m from BC , CD and DA respectively.



(a) Taking the length of AB to be x metres, write down expressions, in terms of x , for

- (i) EF ,
- (ii) BC ,
- (iii) FG .

[2]

(b) Hence show that the area, y square metres, of the lawn, $EFGH$ is given by

$$y = 110 - 5x - \frac{200}{x}. \quad [1]$$

(c) The table below shows some values of x and the corresponding values of y , correct to 1 decimal place, where

$$y = 110 - 5x - \frac{200}{x}.$$

x	4	5	6	7	8	9	10
y	p	45.0	46.7	46.4	45.0	42.8	40.0

Find the value of p .

[1]

(d) Using a scale of 2 cm to 1 metre, draw a horizontal x -axis for $4 \leq x \leq 10$.
 Using a scale of 2 cm to 2 square metres, draw a vertical y -axis for $40 \leq y \leq 48$.
 On your axes, plot the points given in the table and join them with a smooth curve.

[3]

(e) By drawing a tangent, find the gradient of the curve where $x = 8$.

[2]

(f) Use your graph to find

(i) the range of values of x for which the area of the lawn is at least 44 m^2 ,

[2]

(ii) the value of x for which the area of the lawn is greatest.

[1]

5 Answer the whole of this question on a sheet of graph paper.

The table below gives some values of x and the corresponding values of y , correct to one decimal place, where

$$y = \frac{x^2}{8} + \frac{18}{x} - 5.$$

x	1	1.5	2	2.5	3	4	5	6	7	8
y	13.1	7.3	4.5	3.0	2.1	1.5	1.7	p	3.7	5.3

(a) Find the value of p . [1]

(b) Using a scale of 2 cm to 1 unit, draw a horizontal x -axis for $0 \leq x \leq 8$.

Using a scale of 1 cm to 1 unit, draw a vertical y -axis for $0 \leq y \leq 14$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to find

(i) the value of x when $y = 8$, [1]

(ii) the least value of $\frac{x^2}{8} + \frac{18}{x}$ for values of x in the range $0 \leq x \leq 8$. [1]

(d) By drawing a tangent, find the gradient of the curve at the point where $x = 2.5$. [2]

(e) On the axes used in part **(b)**, draw the graph of $y = 12 - x$. [2]

(f) The x coordinates of the points where the two graphs intersect are solutions of the equation

$$x^3 + Ax^2 + Bx + 144 = 0.$$

Find the value of A and the value of B . [2]

6 Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation

$$y = \frac{x^2}{5} + \frac{5}{x}.$$

The table below shows some values of x and the corresponding values of y correct to 1 decimal place.

x	1	1.5	2	3	4	5	6
y	5.2	3.8	3.3	3.5	4.5	6.0	p

(a) Calculate the value of p . [1]

(b) Using a scale of 2 cm to represent 1 unit on each axis, draw a horizontal x -axis for $0 \leq x \leq 6$ and a vertical y -axis for $0 \leq y \leq 9$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to find the values of x in the range $1 \leq x \leq 6$ for which $\frac{x^2}{5} + \frac{5}{x} - 4 = 0$. [2]

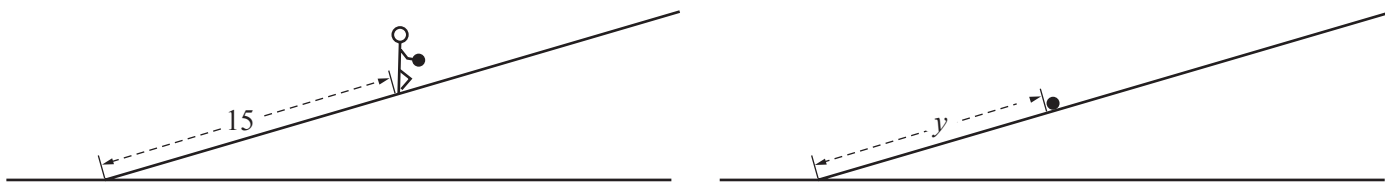
(d) By drawing a tangent, find the gradient of the curve at the point (4, 4.5). [2]

(e) (i) On the same axes, draw the graph of $y = \frac{1}{2}x + 3$. [2]

(ii) Write down the x coordinates of the points at which the two graphs intersect. [1]

(iii) Find the equation, in the form $2x^3 + ax^2 + bx + c = 0$, which is satisfied by the values of x found in part **(e)(ii)**. [1]

7 Answer the whole of this question on a sheet of graph paper.



Adam stood on a slope, 15 m from the bottom.

He rolled a heavy ball directly up the slope.

After t seconds the ball was y metres from the bottom of the slope.

The table below gives some values of t and the corresponding values of y .

t	0	1	2	2.5	3	3.5	4	4.5	5	5.5
y	15	22	25	25	24	22	19	15	10	4

- (a)** Using a scale of 2 cm to represent 1 unit, draw a horizontal t -axis for $0 \leq t \leq 6$.

Using a scale of 2 cm to represent 5 units, draw a vertical y -axis for $0 \leq y \leq 30$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

- (b)** Extend the curve to find the value of t when the ball reached the bottom of the slope. [1]

- (c) (i)** By drawing a tangent, find the gradient of the curve when $t = 3.5$. [2]

- (ii)** State briefly what this gradient represents. [1]

- (d)** Immediately after he rolled the ball, Adam ran down the slope at a constant speed of 1.5 m/s.

- (i)** Write down the distance of Adam from the bottom of the slope when

- (a)** $t = 0$,

- (b)** $t = 4$. [2]

- (ii)** On the same axes, draw the graph that represents the distance of Adam from the bottom of the slope for $0 \leq t \leq 6$. [2]

- (iii)** Hence find the distance of Adam from the bottom of the slope when the ball passed him. [1]

8 Answer the whole of this question on a sheet of graph paper.

A stone was thrown from the top of a vertical cliff. Its position during the flight is represented by the equation $y = 24 + 10x - x^2$, where y metres is the height of the stone above the sea and x metres is the horizontal distance from the cliff.

(a) Solve the equation $0 = 24 + 10x - x^2$. [2]

(b) The table shows some values of x and the corresponding values of y .

x	0	2	4	6	8	10
y	24	40	48	48	40	24

(i) Using a scale of 1 cm to represent 1 metre, draw a horizontal x -axis for $0 \leq x \leq 14$.
Using a scale of 2 cm to represent 10 metres, draw a vertical y -axis for $0 \leq y \leq 50$.

On your axes, plot the points from the table and join them with a smooth curve. [3]

(ii) Use your answer to part **(a)** to complete the graph which represents the flight of the stone. [1]

(iii) Find the height of the stone above the sea when its horizontal distance from the cliff was 7 m. [1]

(iv) Use your graph to find how far the stone travelled horizontally while it was 6 m or more above the top of the cliff. [2]

(c) It is given that $24 + 10x - x^2 = p - (x - 5)^2$.

(i) Find the value of p . [1]

(ii) Hence find

(a) the greatest height of the stone above the sea, [1]

(b) the horizontal distance from the cliff when the stone was at its greatest height. [1]

9 Answer the WHOLE of this question on a sheet of graph paper.

The table below shows some values of x and the corresponding values of y , correct to one decimal place, for

$$y = \frac{4}{5} \times 2^x.$$

x	-2	-1	0	1	2	2.5	3	3.5	4
y	p	0.4	0.8	1.6	3.2	4.5	6.4	9.1	12.8

(a) Calculate p . [1]

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x -axis for $-2 \leq x \leq 4$.

Using a scale of 2 cm to represent 2 units, draw a vertical y -axis for $0 \leq y \leq 14$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) As x decreases, what value does y approach? [1]

(d) By drawing a tangent, find the gradient of the curve at the point (3, 6.4). [2]

(e) (i) On the axes used in part (b), draw the graph of $y = 8 - 2x$. [2]

(ii) Write down the coordinates of the point where the line intersects the curve. [1]

(iii) The x coordinate of this point of intersection satisfies the equation

$$2^x = Ax + B.$$

Find the value of A and the value of B . [2]

10 Answer the whole of this question on a sheet of graph paper.

The heights of 120 children were measured.

The results are summarised in the table below.

Height (h cm)	$135 < h \leq 140$	$140 < h \leq 145$	$145 < h \leq 150$	$150 < h \leq 155$	$155 < h \leq 160$	$160 < h \leq 180$
Frequency	15	20	25	30	20	10

(a) Using a scale of 1 cm to represent 5 cm, draw a horizontal axis for heights from 135 cm to 180 cm. Using a scale of 2 cm to represent 1 unit, draw a vertical axis for frequency densities from 0 to 6 units. On your axes, draw a histogram to represent the information in the table. [3]

(b) Estimate how many children have heights greater than 170 cm. [1]

(c) One child was chosen at random.

Find the probability that the height of this child was less than or equal to 140 cm.

Give your answer as a fraction in its lowest terms. [1]

(d) Two children were chosen at random.

Find the probability that they both had heights in the range $150 < h \leq 155$. [2]

11 Answer the whole of this question on a sheet of graph paper.

The number of bacteria in a colony doubles every half hour.

The colony starts with 50 bacteria.

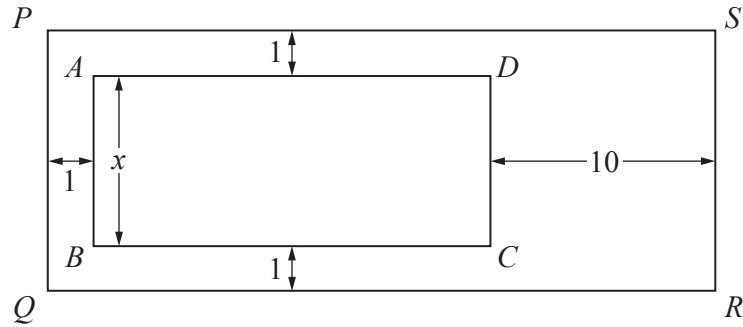
The table below shows the number of bacteria in the colony after t hours.

Time (t hours)	0	0.5	1	1.5	2	2.5	3	3.5
Number of bacteria (y)	50	100	200	400	800	1600	3200	6400

- (a) Using a scale of 4 cm to represent 1 hour, draw a horizontal t -axis for $0 \leq t \leq 4$.
 Using a scale of 2 cm to represent 1000 bacteria, draw a vertical y -axis for $0 \leq y \leq 7000$.
 On your axes, plot the points given in the table, and join them with a smooth curve. [3]
- (b) Use your graph to find the number of bacteria in the colony when $t = 2.75$. [1]
- (c) (i) By drawing a tangent, find the gradient of the graph when $t = 2.5$. [2]
 (ii) State briefly what this gradient represents. [1]
- (d) The number of bacteria in another colony is given by the equation

$$y = 6500 - 1000t.$$
- (i) On the same axes, draw a graph to represent the number of bacteria in this colony. [2]
 (ii) Find the value of t when the number of bacteria in each colony is the same. [1]
- (e) Given that the equation of the first graph is $y = ka^t$, find the value of
- (i) k , [1]
 (ii) a . [1]
-

12 Answer THE WHOLE of this question on a sheet of graph paper.



The diagram represents a rectangular pond, $ABCD$, surrounded by a paved region. The paved region has widths 1 m and 10 m as shown. The pond and paved region form a rectangle $PQRS$. The area of the pond is 168 m^2 .

(a) Taking the length of AB to be x metres, write down expressions, in terms of x , for

(i) PQ ,

(ii) BC ,

(iii) QR .

[2]

(b) Hence show that the area, y square metres, of the paved region, is given by

$$y = 22 + 11x + \frac{336}{x}.$$

[2]

(c) The table below shows some values of x and the corresponding values of y .

x	3	3.5	4	5	6	7	8	9
y	167	156.5	150	144.2	144	147	152	p

Calculate p .

[1]

(d) Using a scale of 2 cm to represent 1 metre, draw a horizontal x -axis for $3 \leq x \leq 9$.

Using a scale of 2 cm to represent 5 square metres, draw a vertical y -axis for $140 \leq y \leq 170$.

On your axes, plot the points given in the table and join them with a smooth curve.

[3]

(e) By drawing a tangent, find the gradient of the curve at $(4, 150)$.

[2]

(f) Use your graph to find

(i) the smallest area of the paved region,

[1]

(ii) the length of PQ when the area of the paved region is smallest.

[1]

13 Answer the whole of this question on a sheet of graph paper.

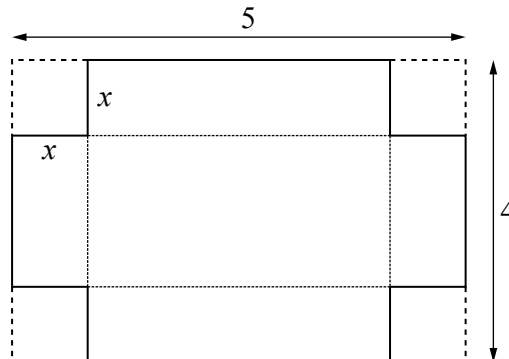
- (a) The variables x and y are connected by the equation

$$y = 4x^3 - 18x^2 + 20x.$$

The table below shows some values of x and the corresponding values of y .

x	0	0.5	1	1.5	2	2.5	3	3.5
y	0	6	6	3	0	0	6	p

- (i) Calculate the value of p . [1]
- (ii) Using a scale of 2 cm to represent 1 unit, draw a horizontal x -axis for $0 \leq x \leq 4$.
Using a scale of 1 cm to represent 2 units, draw a vertical y -axis for $-4 \leq y \leq 24$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]
- (iii) Using your graph, find the values of x when $y = 4$. [2]
- (b) A rectangular card is 5 cm long and 4 cm wide.
As shown in the diagram, a square of side x centimetres is cut off from each corner.



The card is then folded to make an open box of height x centimetres.

- (i) Write down expressions, in terms of x , for the length and width of the box. [1]
- (ii) Show that the volume, V cubic centimetres, of the box is given by the equation
- $$V = 4x^3 - 18x^2 + 20x. \quad [2]$$
- (iii) Which value of x found in (a)(iii) **cannot** be the height of a box with a volume of 4 cm^3 ? [1]
- (iv) Using the graph drawn in part (a)(ii), find
- (a) the greatest possible volume of a box made from this card, [1]
- (b) the height of the box with the greatest volume. [1]
-

14 Answer the WHOLE of this question on a sheet of graph paper.

The table below shows some values of x and the corresponding values of y for

$$y = \frac{12}{x} - x.$$

x	1	2	3	4	5	6
y	11	4	1	-1	p	-4

(a) Calculate p . [1]

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x -axis for $0 \leq x \leq 6$.

Using a scale of 1 cm to represent 1 unit, draw a vertical y -axis for $-4 \leq y \leq 14$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to solve the equation $\frac{12}{x} - x = 2$ in the range $1 \leq x \leq 6$. [1]

(d) The equation $\frac{12}{x} = 2x$ can be solved using the intersection of your curve and a straight line.

(i) State the equation of this straight line. [1]

(ii) By drawing this straight line, solve the equation $\frac{12}{x} = 2x$. [2]

(e) The points A and B are $(1, 11)$ and $(4, -1)$ respectively.

Find the gradient of the line AB . [1]

(f) The line l is parallel to AB and is a tangent to the curve $y = \frac{12}{x} - x$.

(i) Draw the line l . [1]

(ii) Find the coordinates of the point where l crosses the y -axis. [1]

(iii) Hence find the equation of the line l . [1]

15 Answer the WHOLE of this question on a sheet of graph paper.

The table below shows some values of x and the corresponding values of y for

$$y = \frac{2^x}{4}.$$

x	-1	0	1	2	3	4	5
y	m	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	n

- (a) Calculate the values of m and n . [2]

- (b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x -axis for $-1 \leq x \leq 5$.
Using a scale of 2 cm to represent 1 unit, draw a vertical y -axis for $0 \leq y \leq 8$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

- (c) Use your graph to solve the equations

(i) $\frac{2^x}{4} = 3$, [1]

(ii) $2^x = 6$. [1]

- (d) The equation $y = \frac{2^x}{4}$ can be written in the form $y = 2^t$.

- (i) Find an expression for t in terms of x . [1]

- (ii) Hence, find the equation of the line that can be drawn on your graph to evaluate y when $t = -\frac{3}{4}$. [1]

7

16 Answer the WHOLE of this question on a sheet of graph paper.

The variables x and y are connected by the equation

$$y = x^2 - 1.$$

- (a) Copy and complete the following table of values. [2]

x	-4	-3	-2	-1	0	1	2	3	4
y									

- (b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x -axis for $-4 \leq x \leq 4$.
Using a scale of 1 cm to represent 1 unit, draw a vertical y -axis for $-2 \leq y \leq 16$.

On your axes, plot the points from your table and join them with a smooth curve. [3]

- (c) It is given that $f(x) = \frac{x+7}{2}$.

- (i) Using the same axes, draw the graph of $y = f(x)$. [2]

- (ii) Find $f^{-1}(3)$. [2]

- (iii) (a) Write down the x coordinates of the two points where the graphs intersect. [1]

- (b) Find the equation that is satisfied by these two values of x .
Express your answer in the form $ax^2 + bx + c = 0$, where a , b and c are integers. [2]

17 Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation

$$y = \frac{x^3}{10} - \frac{x}{2} = \frac{(4.5)^3}{10} - \frac{4.5}{2}$$

The table below shows some corresponding values of x and y .

x	0	1	2	3	4	4.5
y	0	-0.4	-0.2	1.2	4.4	p

(a) Calculate p . [1]

(b) Using a scale of 2 cm to 1 unit on each axis, draw a horizontal x -axis for $0 \leq x \leq 5$ and a vertical y -axis for $-1 \leq y \leq 7$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to solve the equation $\frac{x^3}{10} - \frac{x}{2} = 0.3$ for values of x in the range $0 \leq x \leq 5$. [1]

(d) (i) Draw the chord joining the two points (0, 0) and (3, 1.2) and calculate its gradient. [1]

(ii) Draw a tangent at the point where the gradient of the curve is equal to the gradient of the chord. [1]

(e) (i) On the same axes, draw the graph of the straight line $y = -x + 6$. [2]

(ii) Write down the x coordinate of the point where the line crosses the curve. [1]

(iii) This value of x is a solution of the equation $x^3 + Ax + B = 0$.

Find A and B . [2]

18 (a)

$$f(x) = \frac{2x+7}{3}$$

(i) Find $f^{-1}(x)$.

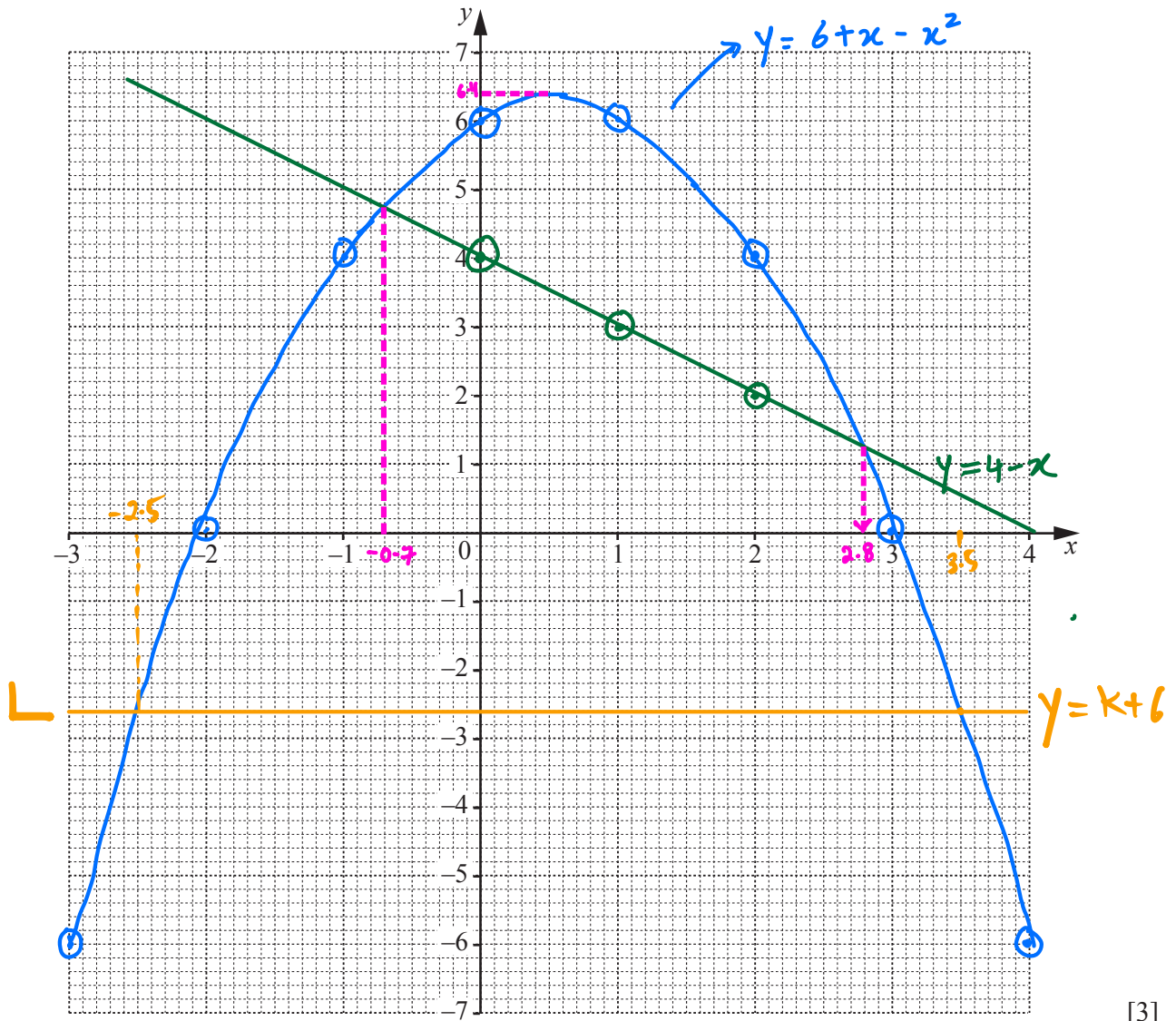
$$\begin{aligned} y &= \frac{2x+7}{3} \\ 3y &= 2x+7 \\ x &= \frac{3y-7}{2} \\ f^{-1}(x) &= \frac{3x-7}{2} \end{aligned}$$

Answer $f^{-1}(x) = \dots\dots\dots$ [2](ii) Given that $f(m) = \frac{m}{2}$, find m .

$$\begin{aligned} \frac{2m+7}{3} &= \frac{m}{2} \\ 4m+14 &= 3m \\ 4m-3m &= -14 \\ m &= -14 \end{aligned}$$

Answer $\dots\dots\dots$ [2](b) (i) Complete the table of values for $y = 6 + x - x^2$, and hence draw the graph of $y = 6 + x - x^2$ on the grid opposite.

x	-3	-2	-1	0	1	2	3	4
y	-6	0	4	6	6	4	0	-6



[3]

- (ii) Use your graph to estimate the maximum value of $6 + x - x^2$.

$$y = 4 - x$$

Answer 6.4 [1]

- (iii) By drawing the line $x + y = 4$, find the approximate solutions to the equation

ORIGINAL
 $y = 6 + x - x^2$

$$y = 4 - x$$

x	0	1	2
y	4	3	2

$$\begin{aligned} 2 + 2x - x^2 &= 0 \\ +4 - x &= 4 - x \\ \hline 6 + x - x^2 &= 4 - x \\ \downarrow \\ y &= 4 - x \end{aligned}$$

Answer $x =$ -0.7 or 2.8 [2]

- (iv) The equation $x - x^2 = k$ has a solution $x = 3.5$.

By drawing a suitable line on the grid, find the other solution.
Label your line with the letter L.

ORIGINAL
 $y = 6 + x - x^2$

$$x - x^2 = k$$

+6

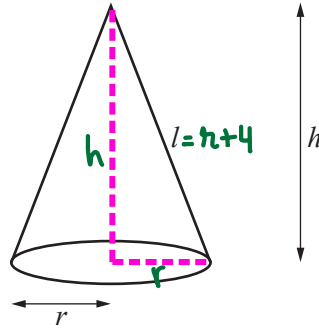
+6

$$6 + x - x^2 = k + 6$$

Answer -2.5 [2]

$$y = k + 6 \text{ (Horizontal line)}$$

- 19 [Curved surface area of a cone = πrl]



$$h^2 + r^2 = (r + 4)^2$$

$$\cancel{h^2 + r^2} = \cancel{r^2} + 8r + 16$$

$$h^2 = 8r + 16$$

$$h = \sqrt{8r + 16}$$

The diagram shows a solid cone with radius r cm, height h cm and slant height l cm. Suleman makes some solid cones.

The slant height of each of his cones is 4 cm more than its radius.

Use $\pi = 3$ throughout this question.

- (a) Show that the **total** surface area, A cm², of each of Suleman's cones is given by $A = 6r(r + 2)$.

$$\begin{aligned}
 A &= \text{Base} + \text{Curved SA} \\
 &= \pi r^2 + \pi rl \\
 &= \pi r^2 + \pi r(r + 4) \\
 &= 3r^2 + 3r(r + 4) \\
 &= 3r^2 + 3r^2 + 12r \\
 &= 6r^2 + 12r \\
 &= 6r(r + 2)
 \end{aligned}$$

[2]

- (b) Complete the table for $A = 6r(r + 2)$.

r	0	1	2	3	4	5	6
A	0	18	48	90	144	210	288

[1]

- (c) On the grid opposite, draw the graph of $A = 6r(r + 2)$.

[2]

- (d) Find an expression for h in terms of r .

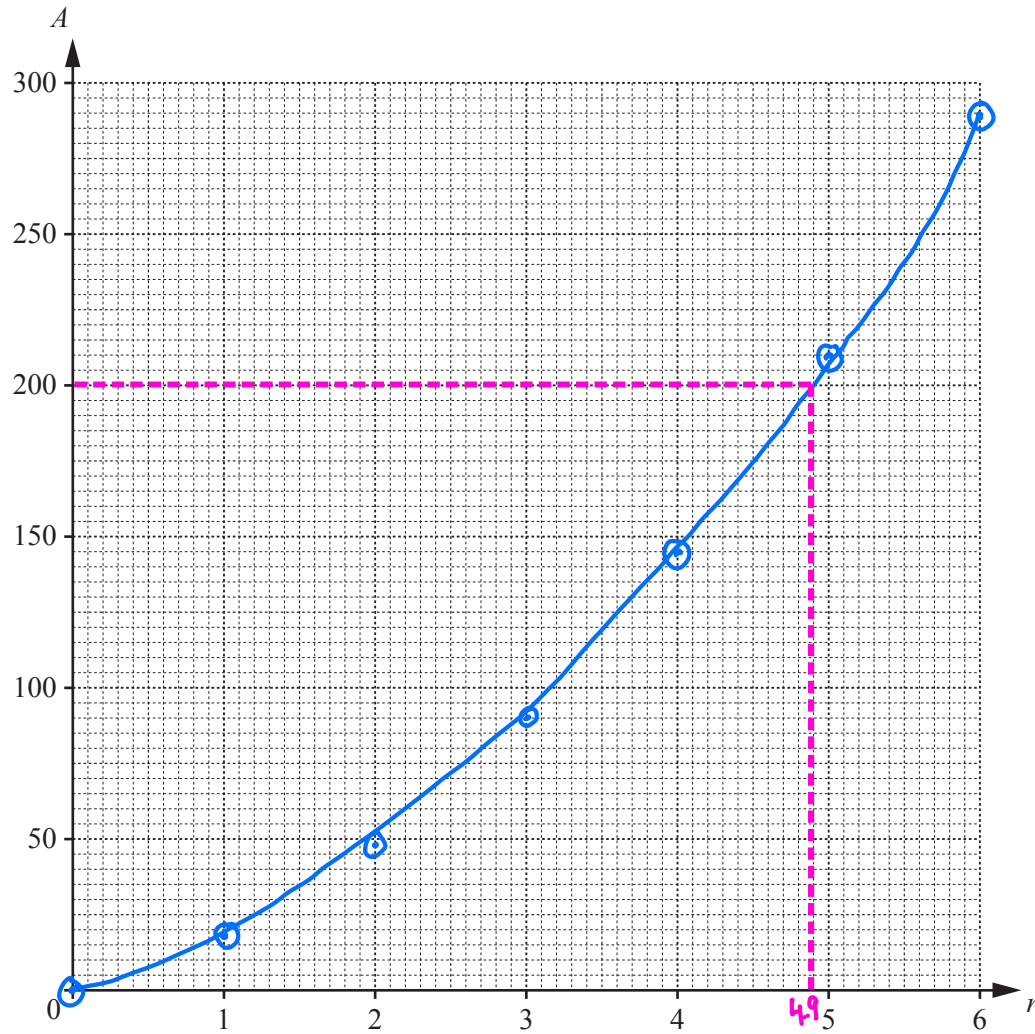
$$h^2 + r^2 = (r + 4)^2$$

$$\cancel{h^2 + r^2} = \cancel{r^2} + 8r + 16$$

$$h^2 = 8r + 16$$

$$h = \sqrt{8r + 16}$$

Answer $h = \sqrt{8r + 16}$ [2]



- (e) The height of one of Suleman's cones is 12 cm.
Calculate its radius.

$$h = \sqrt{8r + 16}$$

$$(12)^2 = (\sqrt{8r + 16})^2$$

$$144 = 8r + 16$$

$$8r = 128$$

$$r = 16$$

Answer 16 cm [2]

- (f) Another of Suleman's cones has a surface area of 200 cm^2 .

- (i) Use your graph to find the radius of this cone.

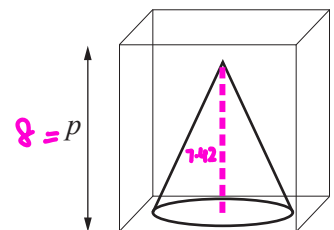
Answer 4.9 cm [1]

- (ii) This cone is placed in a box of height p cm, where p is an integer.
Find the smallest possible value of p .

$$h = \sqrt{8r + 16}$$

$$h = \sqrt{8(4.9) + 16}$$

$$h = 7.43$$



Answer $p =$ 8 [2]

- 20 The distance, d metres, of a moving object from an observer after t minutes is given by

$$d = t^2 + \frac{48}{t} - 20.$$

$$(7)^2 + 48 \div (7) - 20 = 35.857$$

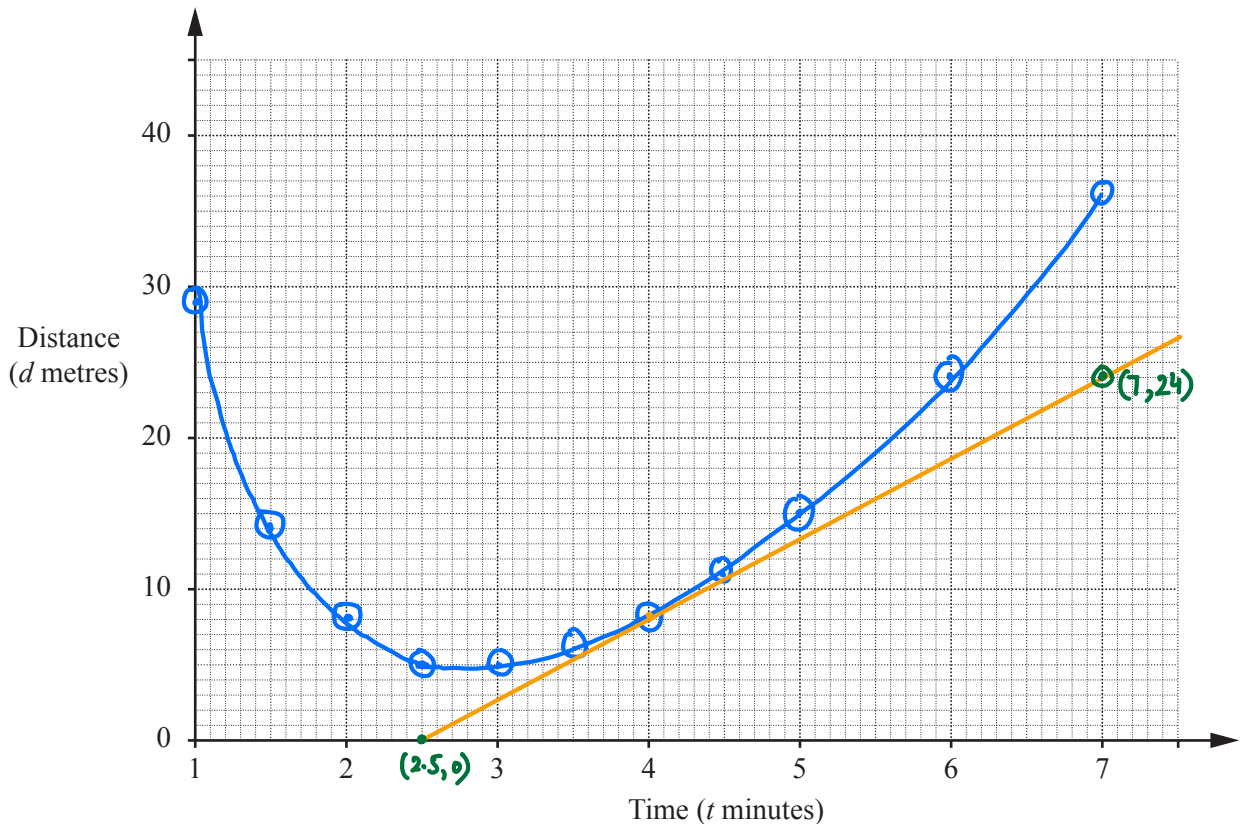
- (a) Some values of t and d are given in the table.
The values of d are given to the nearest whole number where appropriate.

t	1	1.5	2	2.5	3	3.5	4	4.5	5	6	7
d	29	14	8	5	5	6	8	11	15	24	36

Complete the table.

[1]

- (b) On the grid, plot the points given in the table and join them with a smooth curve.



[2]

- (c) (i) By drawing a tangent, calculate the gradient of the curve when $t = 4$.

$$\begin{array}{cc} (2.5, 0) & (7, 24) \\ x_1 & x_2 \end{array}$$

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{24 - 0}{7 - 2.5} = \frac{24}{4.5} = 5.333$$

Answer [2]

- (ii) Explain what this gradient represents.

Answer Rate of change of distance per unit change in time. [1]

- (d) For how long is the object less than 10 metres from the observer?

$$4.3 - 1.8 = 2.5$$

Answer 2.5 minutes [2]

- (e) (i) Using your graph, write down the two values of t when the object is 12 metres from the observer.

For each value of t , state whether the object is moving towards or away from the observer.

Answer When $t = \underline{1.6}$, the object is moving towards the observer.

When $t = \underline{4.6}$, the object is moving away the observer. [2]

- (ii) Write down the equation that gives the values of t when the object is 12 metres from the observer.

$$d = t^2 + \frac{48}{t} - 20$$

$$12 = t^2 + \frac{48}{t} - 20$$

Answer $12 = t^2 + \frac{48}{t} - 20$ [1]

- (iii) This equation is equivalent to $t^3 + At + 48 = 0$.

Find A .

$$d = t^2 + \frac{48}{t} - 20$$

$$12 = t^2 + \frac{48}{t} - 20$$

$$32 = t^2 + \frac{48}{t}$$

$$32 - t^2 = \frac{48}{t}$$

$$32t - t^3 = 48$$

$$0 = t^3 - 32t + 48$$

$$t^3 - 32t + 48 = 0$$

$$t^3 + At + 48 = 0$$

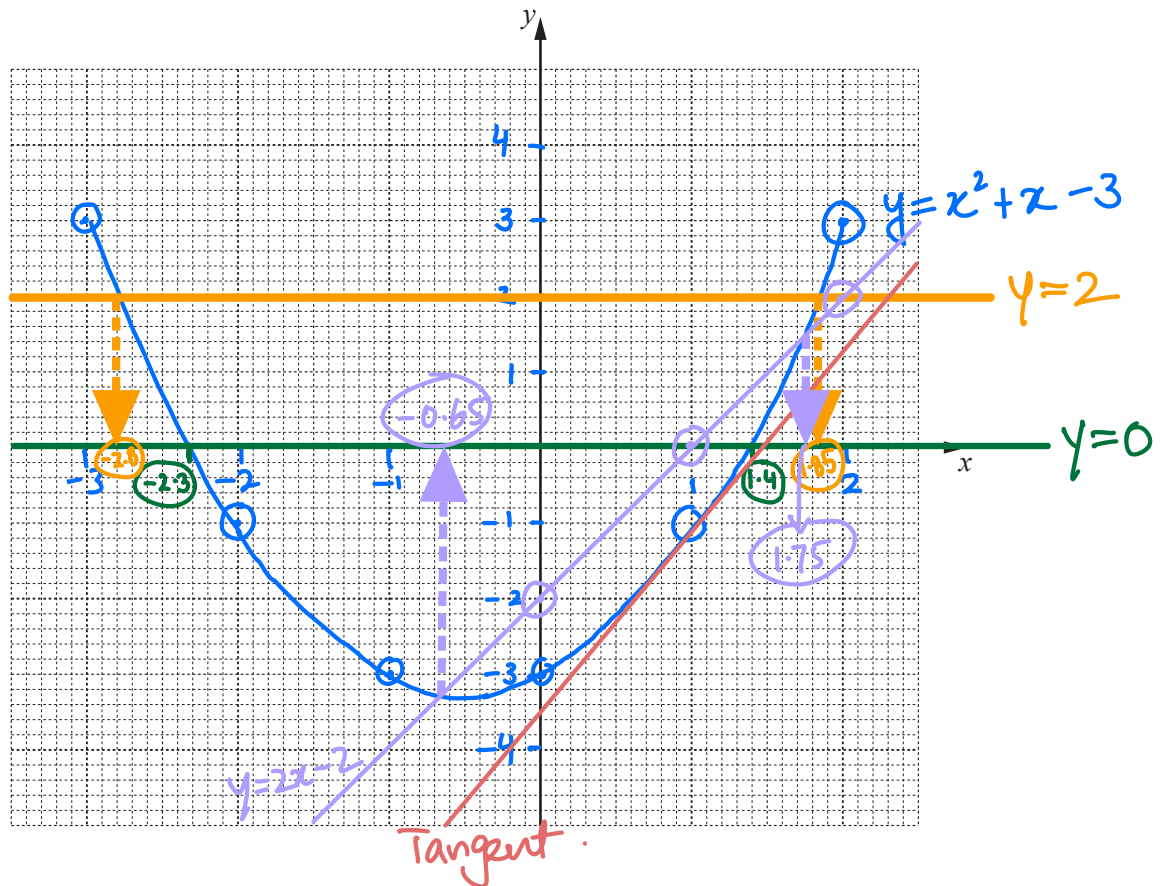
$$A = -32$$

Answer $A = \underline{-32}$ [1]

- 21 The table below is for $y = x^2 + x - 3$.

x	-3	-2	-1	0	1	2
y	3	-1	-3	-3	-1	3

- (a) Using a scale of 2 cm to 1 unit on the x -axis for $-3 \leq x \leq 2$ and a scale of 1 cm to 1 unit on the y -axis for $-4 \leq y \leq 4$, plot the points from the table and join them with a smooth curve.



[2]

Case 1

- (b) (i) Use your graph to estimate the solutions of the equation $x^2 + x - 3 = 0$.

ORIGINAL
 $y = x^2 + x - 3$

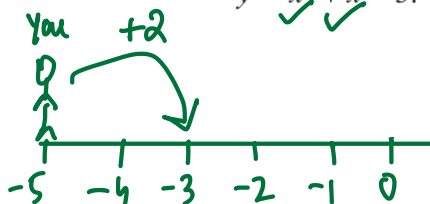
$y = 0$ (Plot) (Horizontal)

Answer $x = \dots 1.4 \dots$ or $\dots -2.3 \dots$ [1]

- (ii) Use your graph to estimate the solutions of the equation $x^2 + x - 5 = 0$.

ORIGINAL
 $y = x^2 + x - 3$

$x^2 + x - 3 = 2$
 $y = 2$ (Horizontal)



Answer $x = \dots -2.8 \dots$ or $\dots 1.8 \dots$ [2]

- (c) By drawing a tangent, estimate the gradient of the curve at $(1, -1)$.

$$\begin{array}{l} (2, 1.4) \\ (0, -3.6) \end{array} \quad \frac{-3.6 - 1.4}{0 - 2} = \frac{-5}{-2} = 2.5$$

Answer [2]

- (d) The equation $x^2 - x - 1 = 0$ can be solved by drawing a straight line on the graph of $y = x^2 + x - 3$.

- (i) Find the equation of this straight line.

Curve
$y = x^2 + x - 3$

Line
$y = mx + c$

SCARY (NOY)
$x^2 - x - 1 = 0$

Curve = line

$$x^2 + x - 3 = mx + c$$

$$x^2 + x - mx - 3 - c = 0$$

$$x^2 + (1-m)x - 3 - c = 0$$

$$\begin{array}{l} \text{SCARY (NOY)} \rightarrow x^2 - x - 1 = 0 \end{array}$$

$$\begin{array}{l} (1-m)x = -x \\ 1-m = -1 \\ 1 = m-1 \\ m = 2 \end{array} \quad \begin{array}{l} -3-c = -1 \\ -3 = c-1 \\ -3+1 = c \\ c = -2 \end{array}$$

Line:

$$y = mx + c$$

$$y = 2x - 2$$

Answer $y = 2x - 2$ [2]

- (ii) Draw this straight line and hence solve $x^2 - x - 1 = 0$. (SCARY)

$$y = 2x - 2$$

x	0	1	2
y	-2	0	2

Answer $x = -0.65$ or 1.75 [2]

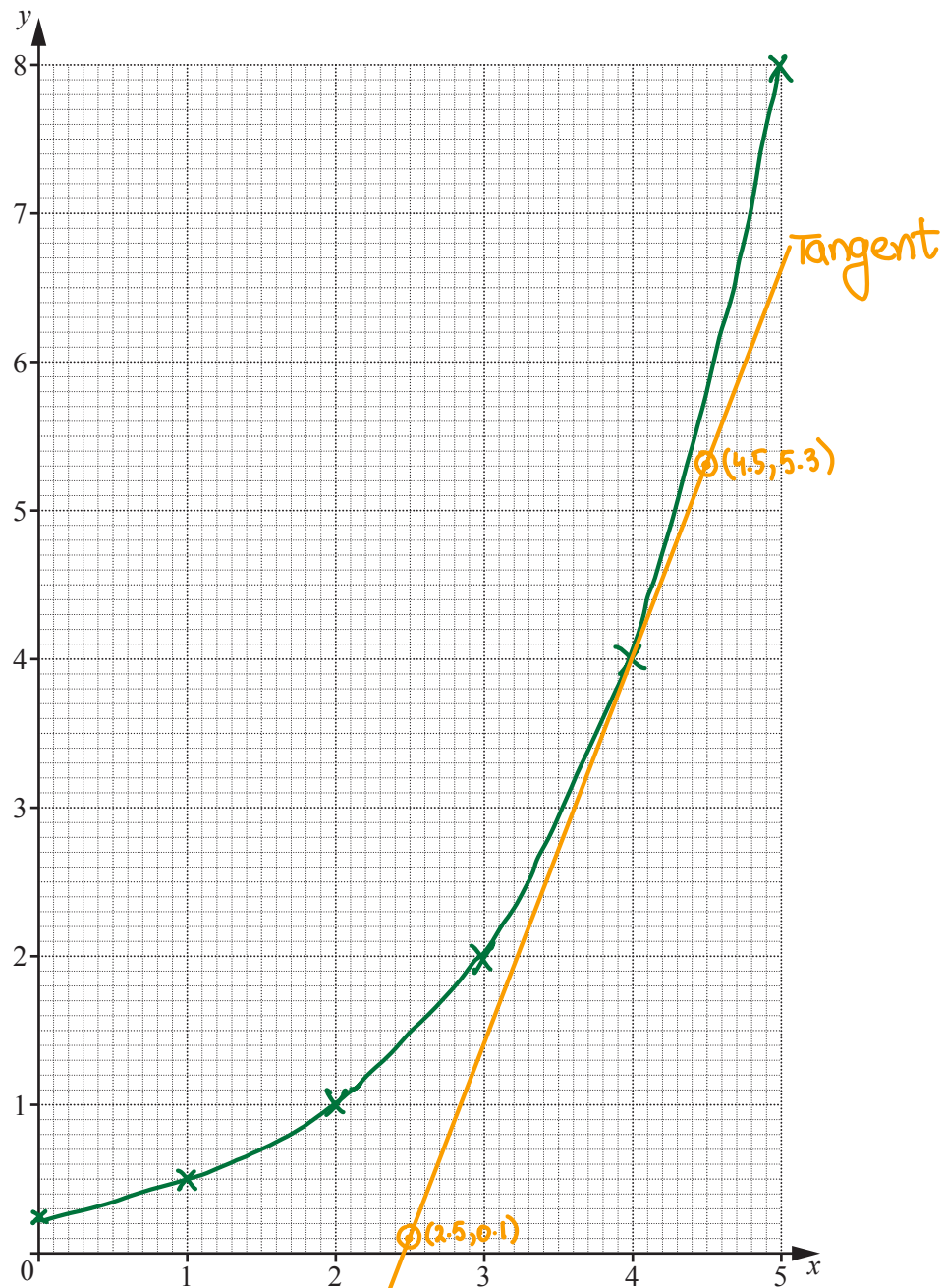
- 22 The table below shows some values of x and the corresponding values of y for $y = \frac{1}{4} \times 2^x$.

x	0	1	2	3	4	5
y	$\frac{1}{4} = 0.25$	0.5	1	2	4	8

(a) Complete the table.

[1]

(b) On the grid below, draw the graph of $y = \frac{1}{4} \times 2^x$. $= \frac{1}{4} \times 2^1 = \frac{1}{2} = 0.5$



[2]

$$\frac{\text{Jump in value}}{\text{Jump in box}} = \frac{1}{10} = 0.1$$

Tangent

(c) By drawing a suitable line, find the gradient of your graph where $x = 4$.

$$\begin{array}{cc} (2.5, 0.1) & (4.5, 5.3) \\ x_1 & x_2 \\ y_1 & y_2 \end{array} \left\{ \begin{array}{l} \text{gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5.3 - 0.1}{4.5 - 2.5} = \frac{5.2}{2} = 2.6 \end{array} \right.$$

Answer 2.6 [2]

(d) (i) Show that the line $2x + y = 6$, together with the graph of $y = \frac{1}{4} \times 2^x$, can be used to solve the equation

$$2^x + 8x - 24 = 0.$$

[1]

(ii) Hence solve $2^x + 8x - 24 = 0$.

Answer $x =$ [2]

(e) The points P and Q are $(2, 3)$ and $(5, 4)$ respectively.

(i) Find the gradient of PQ .

Answer [1]

(ii) On the grid, draw the line l , parallel to PQ , that touches the curve $y = \frac{1}{4} \times 2^x$. [1]

(iii) Write down the equation of l .

Answer [2]

23

$$y = \frac{3}{5} \times 2^x = \frac{3}{5} \times 2^{-1.5} = 0.2 | 2 | = 0.2$$

The table shows some values of x and the corresponding values of y , correct to one decimal place where necessary.

x	-1.5	-1	0	1	2	2.5	3	3.5	4
y	$p=0.2$	0.3	0.6	1.2	2.4	3.4	4.8	6.8	9.6

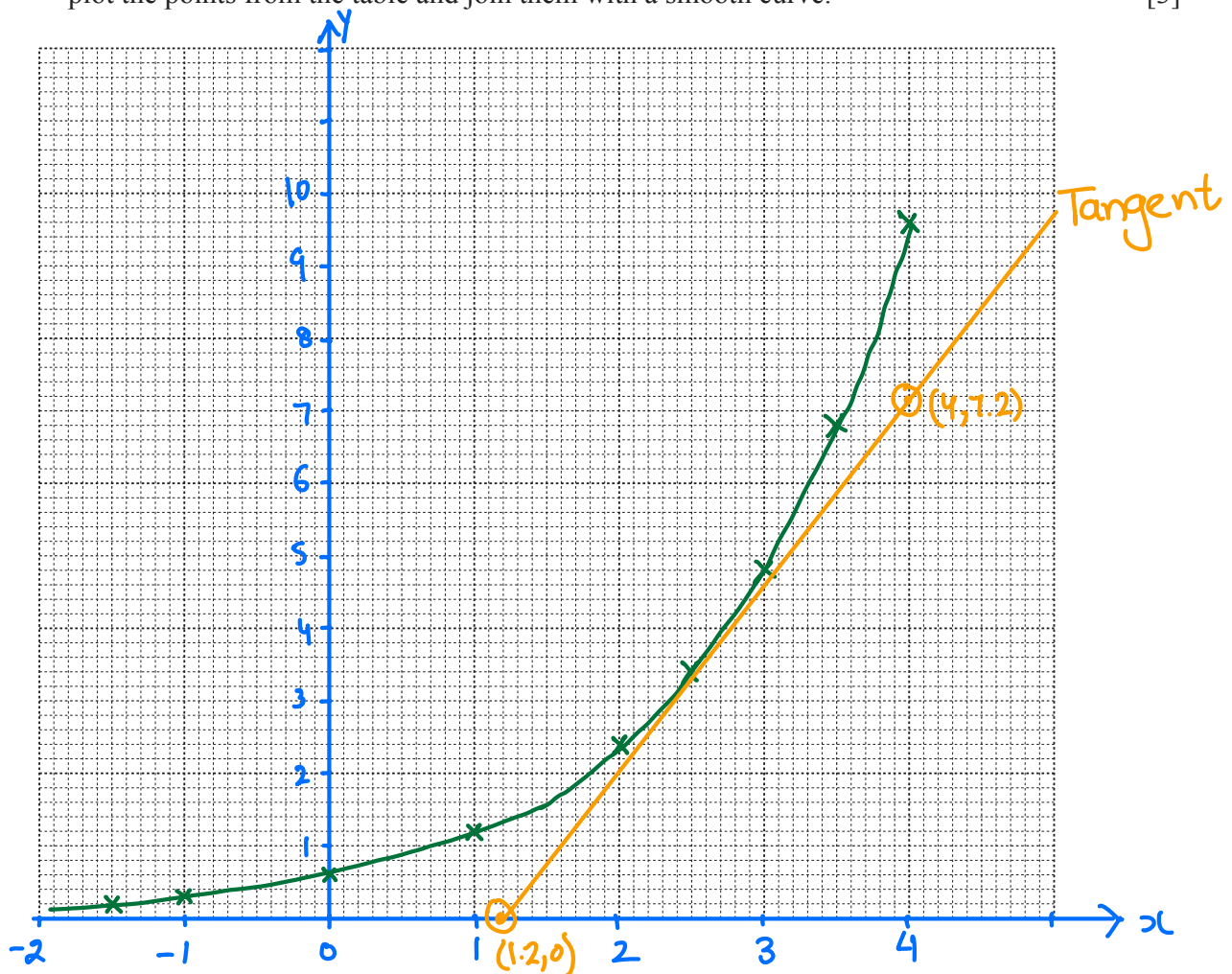
(a) Calculate p .

Answer 0.2 [1]

(b) On the grid,

- using a scale of 2 cm to 1 unit, draw a horizontal x -axis for $-2 \leq x \leq 4$,
- using a scale of 1 cm to 1 unit, draw a vertical y -axis for $0 \leq y \leq 10$,
- plot the points from the table and join them with a smooth curve.

[3]



Value of } x -axis: $\frac{1}{10} = 0.1$
 one small box } y -axis: $\frac{1}{5} = 0.2$

(c) By drawing a tangent, estimate the gradient of the curve at the point where $x = 2.5$.

$$\begin{matrix} (1.2, 0) & (4, 7.2) \\ x_1 & y_1 & x_2 & y_2 \end{matrix} \quad \text{gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7.2 - 0}{4 - 1.2} = \frac{7.2}{2.8} = 2.57$$

Answer 2.57 [2]

(d) (i) On the same grid, draw the straight line that passes through $(-0.4, 0)$ and $(2, 3.6)$.

[1]

(ii) Find the equation of this line in the form $y = mx + c$.

Answer [2]

(iii) Write down the x -coordinates of the points where the line intersects the curve.

Answer $x = \dots\dots\dots$ and $x = \dots\dots\dots$ [1]

(iv) These x -coordinates satisfy the equation

$$2^x = Ax + B.$$

Find the values of A and B .

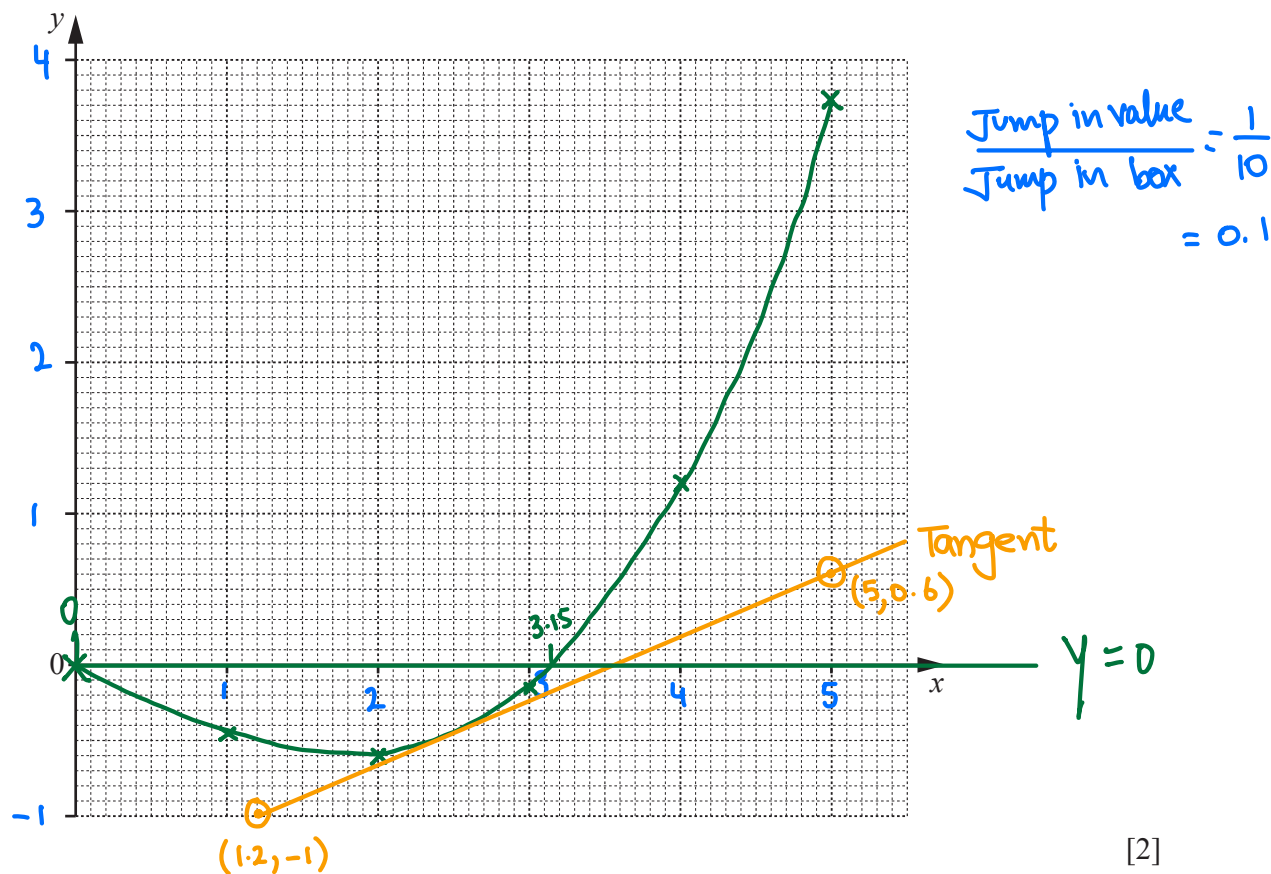
Answer $A = \dots\dots\dots$ $B = \dots\dots\dots$ [2]

- 24 (a) Complete the table of values for $y = \frac{x}{20}(x^2 - 10) = \frac{(5)}{20}(5^2 - 10) =$

x	0	1	2	3	4	5
y	0	-0.45	-0.6	-0.15	1.2	3.15

[1]

- (b) Using a scale of 2 cm to 1 unit on both axes, draw the graph of $y = \frac{x}{20}(x^2 - 10)$ for $0 \leq x \leq 5$.



[2]

- (c) By drawing a tangent, estimate the gradient of the curve at the point where $x = 2.5$.

$$\begin{matrix} (1.2, -1) & (5, 0.6) \\ x_1 & x_2 \\ y_1 & y_2 \end{matrix} \left\{ \text{gradient} = \frac{0.6 - (-1)}{5 - 1.2} = 0.42 \right.$$

Answer [2]

$$y = \frac{x}{20}(x^2 - 10)$$

- (d) Use your graph to solve the equation

ORIGINAL

$$\frac{x}{20}(x^2 - 10) = 0 \quad \text{for } 0 \leq x \leq 5.$$

$y = 0$ (Horizontal)

Answer $x = 0$ or 3.15 [2]

- (e) The graph of $y = \frac{x}{20}(x^2 - 10)$, together with the graph of a straight line L , can be used to solve the equation $x^3 + 10x - 80 = 0$ for $0 \leq x \leq 5$.

- (i) Find the equation of line L .

Answer [2]

- (ii) Draw the graph of line L on the grid. [1]

- (iii) Hence solve the equation $x^3 + 10x - 80 = 0$ for $0 \leq x \leq 5$.

Answer $x =$ [1]

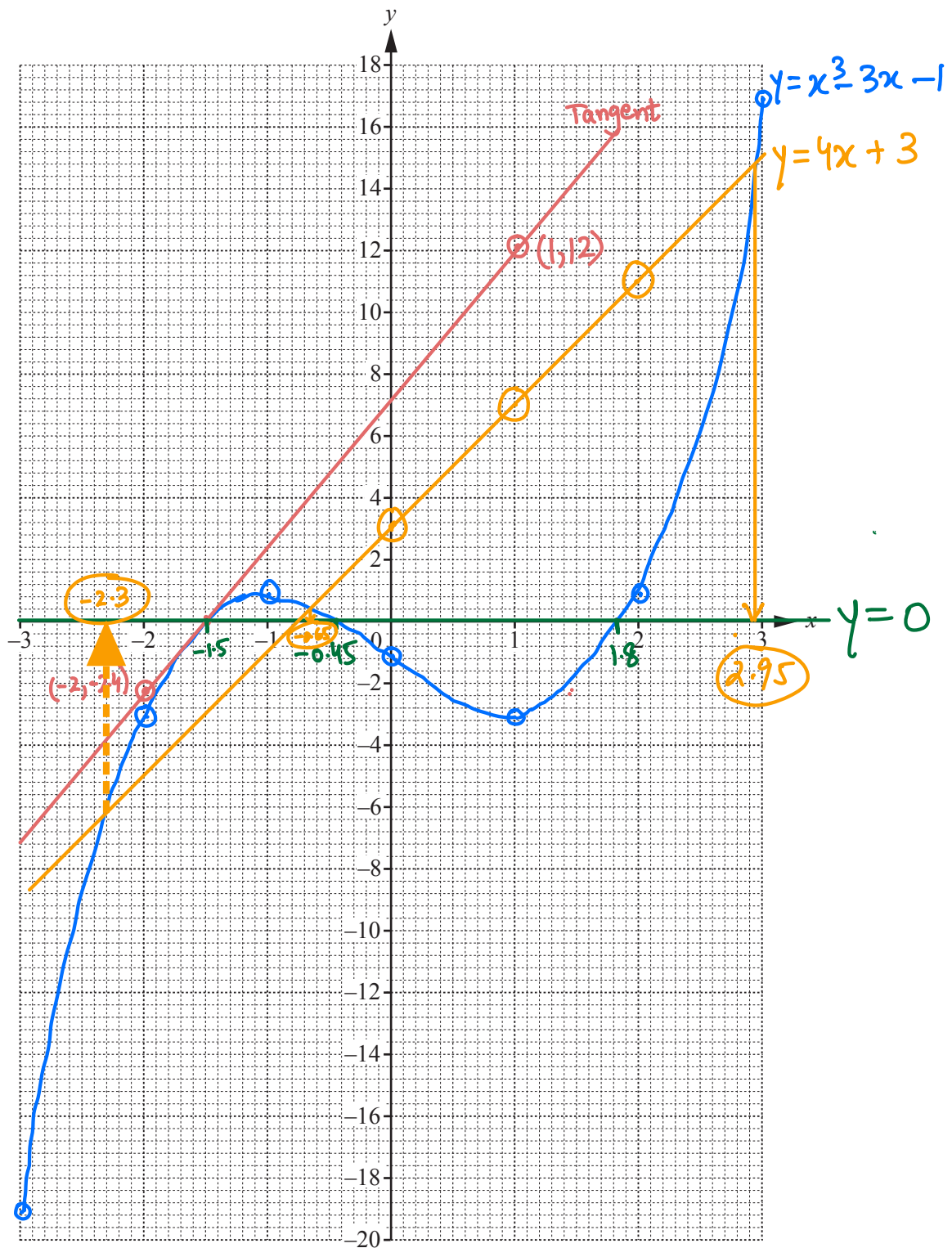
25 The table below is for $y = x^3 - 3x - 1$.

x	-3	-2	-1	0	1	2	3
y	-19	-3	1	-1	-3	1	17

(a) Complete the table.

[1]

(b) On the grid, draw the graph of $y = x^3 - 3x - 1$.



[3]

$$y = x^3 - 3x - 1$$

Case 1

(c) Use your graph to solve $x^3 - 3x - 1 = 0$.

✓ ✓ ✓
 $y = 0$ (horizontal)

Answer $x = -1.5, -0.45, 1.8$ [2]

(d) Use your graph to estimate the gradient of the curve when $x = -1.5$.

(1, 12) (-2, -2.4)

$$\text{gradient} = \frac{-2.4 - 12}{-2 - 1} = \frac{-14.4}{-3} = 4.8$$

Answer [2]

(e) (i) On the grid draw the graph of $y = 4x + 3$.

$$\begin{aligned} 4(0) + 3 &= 3 \\ 4(1) + 3 &= 7 \\ 4(2) + 3 &= 11 \end{aligned}$$

x	0	1	2
y	3	7	11

[1]

(ii) The line $y = 4x + 3$ and the curve $y = x^3 - 3x - 1$ can be used to solve the equation $x^3 = ax + b$.

Find the values of a and b .

curve = line

$$x^3 - 3x - 1 = 4x + 3$$

$$x^3 = 4x + 3x + 3 + 1$$

$$x^3 = 7x + 4$$

$$x^3 = ax + b \text{ (SCARY EQ)}$$

Answer $a = 7$ $b = 4$ [2]

(iii) Use your graph to find one of the **negative** solutions of $x^3 = ax + b$ (SCARY EQ)

Line: $y = 4x + 3$ (plot)

Solutions = -2.3, -0.65, 2.9
 of SCARY EQ

Answer $x =$ [1]

Curve
 $y = x^3 - 3x - 1$

Line
 $y = 4x + 3$

SCARY (NOY)
 $x^3 = ax + b$

26 A random number, x , is generated, where x is any real number.

- (a) Manuel adds 2 to x .
 He subtracts x from 10.
 Manuel then multiplies these two results to give his number, y .

Show that $y = 20 + 8x - x^2$.

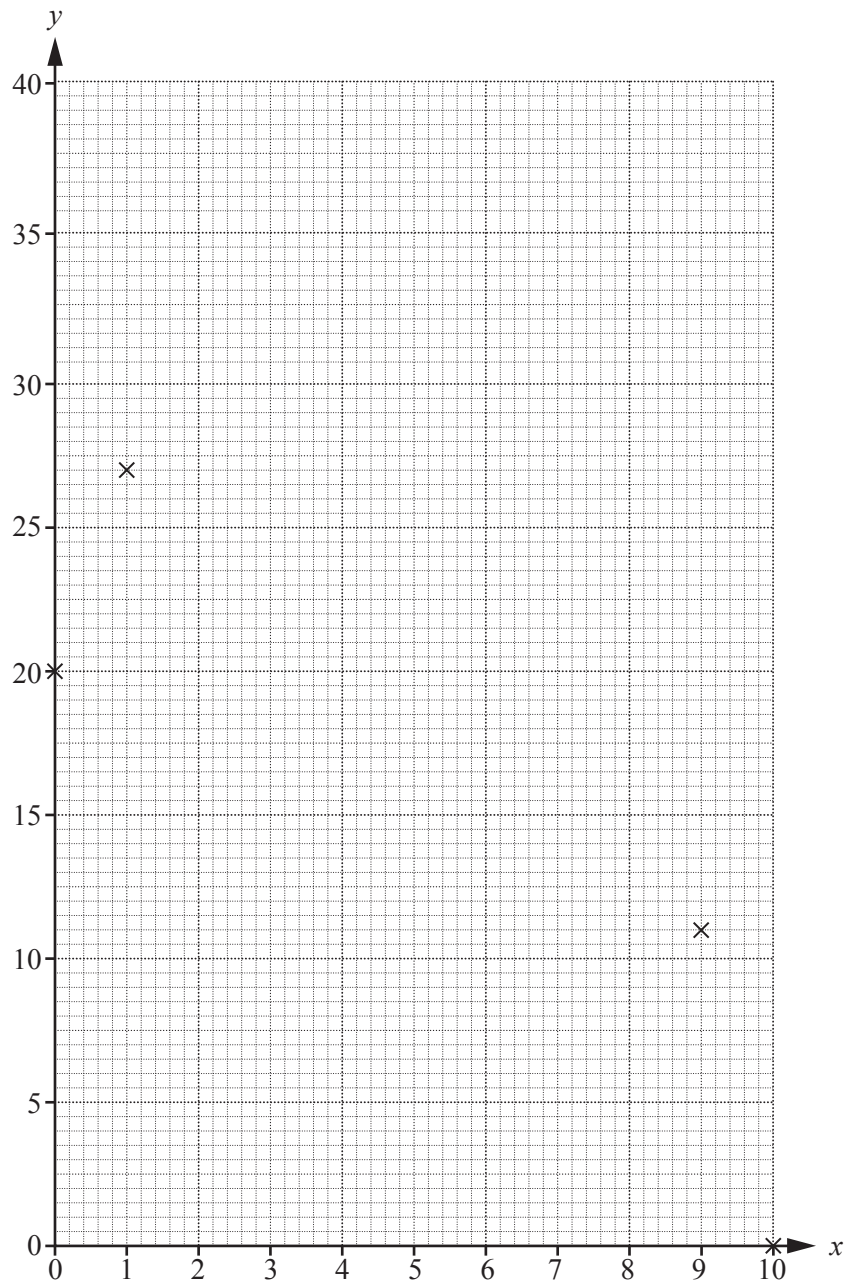
[2]

- (b) On the grid opposite, draw the graph of $y = 20 + 8x - x^2$ for $0 \leq x \leq 10$.
 Four points have been plotted for you.

[4]

- (c) On the same grid, draw a suitable line to find the value of Manuel's number, y , when it is the same as the random number, x .

Answer [2]



- (d) Jolene multiplies the random number, x , by 5 and then adds 2 to give her number, z .

Calculate the possible values of x when Manuel's number, y , and Jolene's number, z , are the same.

Answer $x = \dots\dots\dots$ or $\dots\dots\dots$ [4]

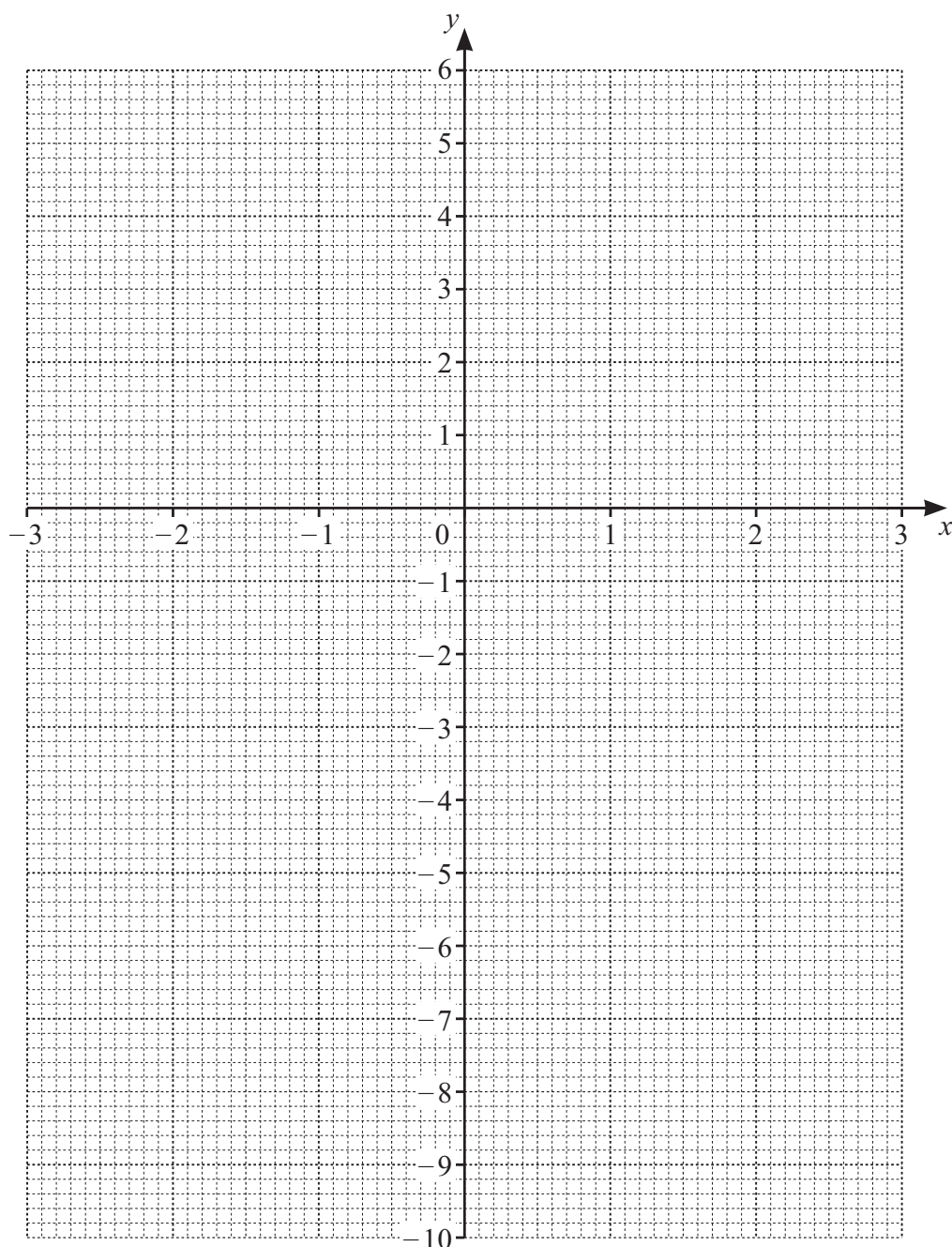


6 (a) Complete the table for $y = \frac{x}{4}(2x^2 - x - 10)$.

x	-3	-2	-1	0	1	2	3
y		0	1.75	0	-2.25	-2	3.75

[1]

(b) Draw the graph of $y = \frac{x}{4}(2x^2 - x - 10)$ for $-3 \leq x \leq 3$.



[3]





- (c) The equation $\frac{x}{4}(2x^2 - x - 10) = k$ has exactly two solutions.

Use your graph to find the possible values of k .

$$k = \dots\dots\dots \text{ or } k = \dots\dots\dots [2]$$

- (d) By drawing a suitable line on the grid, find the solutions of $2x^3 - x^2 - 10x = 2x - 4$.

$$x = \dots\dots\dots, x = \dots\dots\dots, x = \dots\dots\dots [4]$$



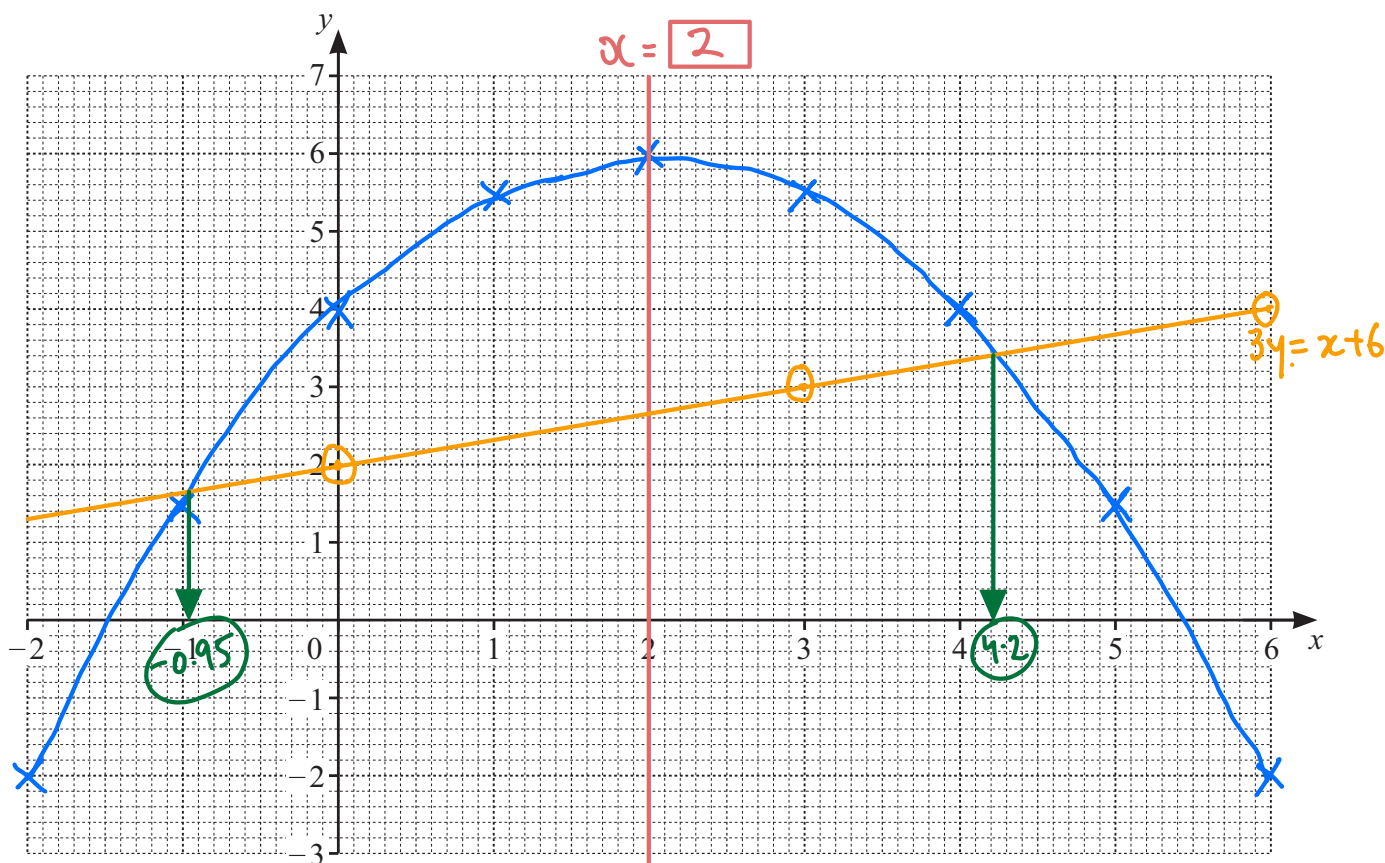


- 3 (a) Complete the table for $y = 4 + 2x - \frac{x^2}{2}$.

x	-2	-1	0	1	2	3	4	5	6
y	-2	1.5	4	5.5	6	5.5	4	1.5	-2

[2]

- (b) Draw the graph of $y = 4 + 2x - \frac{x^2}{2}$ for $-2 \leq x \leq 6$.



[3]

- (c) Find the equation of the line of symmetry of the graph of $y = 4 + 2x - \frac{x^2}{2}$.

..... $x = 2$ [1]

- (d) On the grid, draw the line $3y = x + 6$ for $-2 \leq x \leq 6$.

$$y = \frac{x+6}{3}$$

x	0	3	6
y	2	3	4

[2]

- (e) Write down the x -coordinates of the points of intersection of the graphs of $y = 4 + 2x - \frac{x^2}{2}$ and $3y = x + 6$.

$x = -0.95$ and $x = 4.2$ [1]

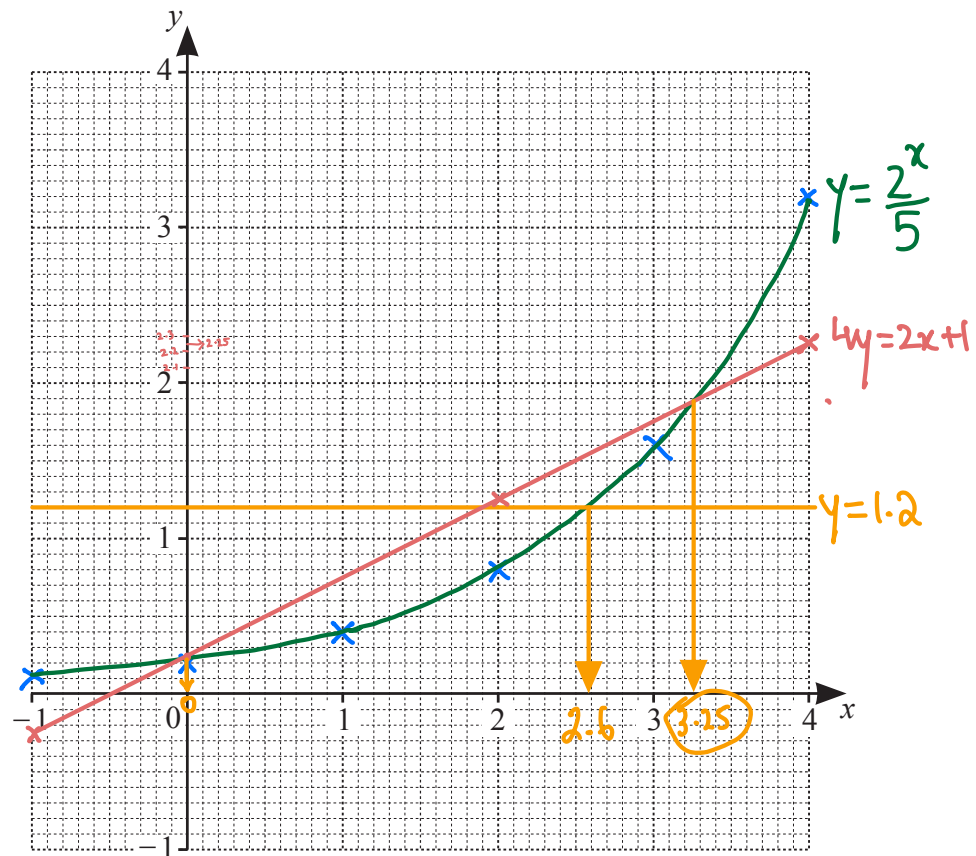


- 4 (a) Complete the table of values for $y = \frac{2^x}{5}$.

x	-1	0	1	2	3	4
y	0.1	0.2	0.4	0.8	1.6	3.2

[1]

- (b) Draw the graph of $y = \frac{2^x}{5}$ for $-1 \leq x \leq 4$.



[3]

- (c) By drawing a suitable line on the grid, solve $\frac{2^x}{5} = \frac{6}{5}$.
 ORIGINAL
 Case 1
 $y = 1.2$ (Horizontal)

$x = 2.6$ [2]

- (d) (i) Complete the table of values for $4y = 2x + 1$.

$$y = \frac{2x+1}{4}$$

x	-1	2	4
y	-0.25	1.25	2.25

[1]

- (ii) On the grid on page 8, draw the graph of $4y = 2x + 1$ for $-1 \leq x \leq 4$. [1]

- (iii) Find the x -coordinates of the points where the line $4y = 2x + 1$ crosses the graph of $y = \frac{2^x}{5}$.

$$x = 0 \text{ and } x = 3.25 \quad [1]$$

- (iv) The x -coordinates in **part (d)(iii)** are the solutions of the equation $A \times 2^x + Bx + C = 0$, where A , B and C are all integers.

Use the equations $4y = 2x + 1$ and $y = \frac{2^x}{5}$ to find the exact value of each of A , B and C .

Curve = Line

$$\frac{2^x}{5} = \frac{2x+1}{4}$$

$$4 \times 2^x = 10x + 5$$

$$4 \times 2^x - 10x - 5 = 0$$

$$A \times 2^x + Bx + C = 0$$

$$\downarrow$$

$$A=4, \quad B=-10, \quad C=-5$$

$$B=-10 \quad C=-5$$

$$A = 4$$

$$B = -10$$

$$C = -5 \quad [3]$$